

## Appendix F Field Reduction and Adjustment of GPS Surveys

### F-1. General

This appendix contains sample data reduction, adjustment, and analysis of GPS surveys. It is intended for guidance to field personnel performing field-to-finish survey work with the GPS. GPS survey data can (and should) be evaluated as soon as possible after observations are completed, preferably within 1 or 2 days. This appendix covers evaluation of internal closures, external closures, adjustment techniques, and evaluation of the adequacy of these results. A PC-based least squares adjustment package is not necessary to perform acceptable final field adjustments. Most USACE GPS work, other than baseline reductions, can be analyzed and adequately adjusted using simple hand-held calculators, as shown herein.

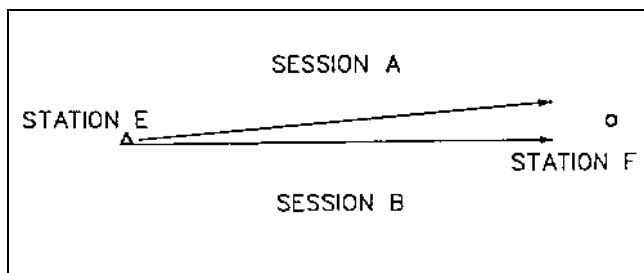


Figure F-1. Spur line adjustment

### F-2. Mean Coordinate Adjustment of Spur Lines

If spur lines are observed twice between points E and F, as shown in Figure F-1, a simple mean adjustment computation is recommended. This method is applicable not only to carrier phase measurements but also code phase positioning techniques. It is important that the user

determine the acceptable closure limits. This evaluation simply involves comparing the differences between multiple sessions taken over the same baseline. Alternately, a double spur line can be considered as a loop, from which the internal loop closure can be computed. Two independent baseline sessions were observed between points E and F, as shown in Table F-1.

The known geocentric coordinates of point E are:

$$\begin{aligned} X &= 1108302.838 \\ Y &= -4856338.733 \\ Z &= 3970134.434 \end{aligned}$$

Computing the 3D misclosure between the vectors:

$$(0.002^2 + (-)0.002^2 + 0.001^2)^{1/2} = 0.003 \text{ m}$$

$$\begin{aligned} \text{3D vector distance} &= (113.841^2 + 44.284^2 \\ &\quad + 18.800^2)^{1/2} = 123.589 \text{ m} \end{aligned}$$

The relative accuracy estimate between the two vectors is then:

$$0.003/123.589 \text{ or } 1:41,200 \text{ (acceptable)}$$

Given the acceptable check between the two observations, the vectors for the two sessions can be simply averaged. Since E is the known station, the mean vector components shown in Table F-1 can be applied to the geocentric coordinates of E to position station F.

Point E adjusted geocentric coordinates:

$$\begin{aligned} X &= 1108302.838 + (-)113.841 = 1108188.997 \\ Y &= -4856338.733 + 44.284 = -4856294.449 \\ Z &= 3970134.434 + 18.800 = 3970153.234 \end{aligned}$$

Final geographic coordinates and/or SPCS coordinates of point F can then be transformed using the techniques

Table F-1  
Baseline Sessions

Vector	Julian Day	Baseline Session	DX m	DY m	DZ m
E-F	135	A	-113.842	44.283	18.800
E-F	135	B	-113.840	44.285	18.799
Vector Differences			0.002	-0.002	0.001
Mean Vector Component			-113.841	44.284	18.800

given in Chapter 11. The position should be identified as a “no check” point as would be done in conventional survey practice.

### F-3. Field Adjustment of GPS Triangle

This example illustrates the various methods which may be used to evaluate the internal and external accuracies of a GPS survey in the field. In addition, both an approximate and rigorous least squares adjustment are performed on the same GPS data to illustrate the small differences in results.

Multiple GPS baseline sessions are observed on the triangle ETLE-HEC2-ETLN, as shown in Figure F-2. Station ETLN is the unknown point for which coordinates are desired to an accuracy of 1 part in 10,000 (Third-Order, Class I). Stations ETLE and HEC2 are fixed, with the following geocentric (WGS 84) metric coordinates:

	X	Y	Z
HEC2	1108302.838	-4856338.733	3970134.434
ETLE	1108314.518	-4856507.916	3969923.835
Diff:	-11.680	169.183	210.599

(The above geocentric coordinates may have been computed in the field using the algorithms given in Chapter 11 on either NAD 83 or NAD 27 datums)

Observed and mean vectors from the baseline reductions are shown in Tables F-2 and F-3.

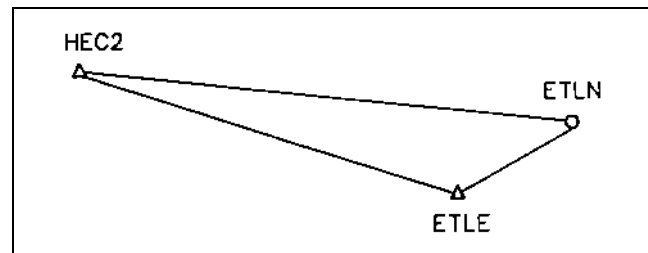


Figure F-2. GPS triangle vector adjustment

### F-4. Internal GPS Loop Closure Check

A loop closure check is performed by arbitrarily letting one set of coordinates equal to zero, then algebraically adding vector components around the loop back to the initial point. Care must be taken in applying the correct vector signs based on the observed vector direction.

Letting station ETLE be fixed ( $X = Y = Z = 0$ ), and using Session A for line ETLE-HEC2 and ETLE-ETLN and Session B for line ETLN-HEC2, and proceeding counter-clockwise around the loop:

$$\Delta x = 98.418 + (-110.083) + (-)(-11.676) = +0.011 \text{ m}$$

$$\Delta y = 9.929 + (-)(-159.250) + (-)169.179 = 0.000 \text{ m}$$

Table F-2  
Observed Vectors from Sessions A and B

Vector	Session	dx	dy	dz
ETLE-ETLN	A	98.418	9.929	-30.837
ETLE-HEC2	A	-11.676	169.179	210.612
ETLN-HEC2	A	-110.094	159.251	241.448
ETLE-ETLN	B	98.405	9.932	-30.834
ETLE-HEC2	B	-11.676	169.184	210.602
ETLN-HEC2	B	-110.083	159.250	241.444

Table F-3  
Mean Vector Components for Sessions A and B

Mean Vector	dx	dy	dz	Distance
ETLE-ETLN	98.412	9.930	-30.836	103.607
ETLE-HEC2	-11.676	169.182	210.607	270.396
ETLN-HEC2	-110.089	159.251	241.446	309.478

$$\Delta z = -30.837 + (-)(-241.444) + (-)210.612 = (-)0.005 \text{ m}$$

(Note that any sequence of session vectors could have been used to perform this check)

$$\begin{aligned} \text{Linear 3D closure} &= (0.011^2 + 0.000^2 + -0.005^2)^{1/2} \\ &= 0.012 \text{ m} \end{aligned}$$

$$\begin{aligned} \text{3D vector distance} &= (103.607 + 270.396 + 309.478) \\ &= 683.5 \text{ m} \end{aligned}$$

Where the individual vector distances were computed by taking the square of the sum of the squares of the component vectors.

The relative accuracy estimate of the loop closure is then:

$$0.012/683.5 \text{ or } 1:57,000 \text{ (acceptable)}$$

This relative accuracy estimate (1 part in 57,000) is based on the internal loop closure results, and indicates that the basic GPS observations are acceptable for subsequent constrained adjustment of station ETLN in the fixed network of HEC2 and ETLE.

#### F-5. Verification of GPS Distances Over Fixed Baselines

The following computation checks the adequacy of the GPS observations over the existing fixed network, i.e., between HEC2 and ETLE. Computing the difference between the mean session vector (from Table F-3) and true vector components over the fixed baseline between ETLE and HEC2:

$$\begin{aligned} \text{Delta X} &= -11.676 - -11.680 = 0.004 \\ \text{Delta Y} &= 169.182 - 169.183 = -0.001 \\ \text{Delta Z} &= 210.607 - 210.599 = 0.008 \end{aligned}$$

The linear misclosure over the baseline is then checked relative to the length of the line:

$$\begin{aligned} \text{Linear 3D closure} &= (0.004^2 + 0.001^2 + 0.008^2)^{1/2} \\ &= 0.009 \text{ m} \end{aligned}$$

The relative accuracy estimate of the baseline closure is then:

$$0.009/270.4 \text{ or } 1:30,000 \quad (\text{OK})$$

This indicates that the observed baseline vector agrees with the fixed control scheme on the order of 1 part in 30,000. Had this check been poor--say only 1 part in 2,500--this would more than likely indicate a problem

with the fixed control network, given the excellent internal loop closures obtained. In such instances, additional fixed control points would have to be connected.

#### F-6. External Closure Verification (GPS Traverse)

This computation illustrates the process for checking the external closure on a GPS traverse run from ETLE to ETLN, and closing on HEC2 (i.e., vector ETLE-ETLN (Session A) and vector ETLN-HEC2 (Session B)). The GPS traverse vectors (Figure F-3) are summed forward as described previously.

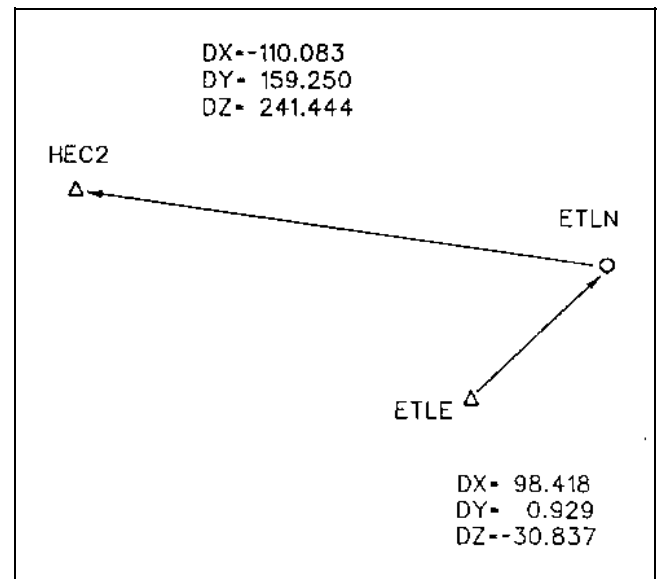


Figure F-3. External traverse closure checks

$$\begin{aligned} X_{\text{HEC2}} &= 1108314.518 + 98.418 + (-110.083) \\ &= 1108302.853 \end{aligned}$$

$$\begin{aligned} Y_{\text{HEC2}} &= -4856507.916 + 9.929 + 159.250 \\ &= -4856338.737 \end{aligned}$$

$$\begin{aligned} Z_{\text{HEC2}} &= 3969923.835 + (-30.837) + 241.444 \\ &= 3970134.442 \end{aligned}$$

Comparing the difference between these computed points and the fixed (i.e., published) points for HEC2:

$$\begin{aligned} \Delta X &= \text{measured/computed coordinate} - \text{true coordinate} \\ &= 1108302.853 - 1108302.838 = 0.015 \text{ m} \\ \Delta Y &= -4856338.737 - (-4856338.733) = (-) 0.004 \text{ m} \\ \Delta Z &= 3970134.442 - 3970134.434 = 0.008 \text{ m} \end{aligned}$$

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The linear misclosure at the traverse closing point (HEC2) then checked relative to the total length of the traverse. This is performed similarly to conventional traverses except that three dimensions and no azimuth misclosures are involved:

$$\begin{aligned}\text{Linear 3D closure} &= (0.015^2 + 0.004^2 + 0.008^2)^{1/2} \\ &= 0.017 \text{ m}\end{aligned}$$

$$3\text{D traverse length} = 103.6 + 309.5 = 413.1 \text{ m}$$

The relative accuracy estimate of the absolute (external) traverse closure is then:

$$0.017/413.1 \text{ or } 1:24,000 \quad (\text{OK})$$

This result is consistent with the previous check over the fixed baseline ETLE-HEC2 and the internal loop closure results. (In practice, GPS traverses will have more legs than this example, and a GPS observation may not have been made between fixed network points.)

The misclosures at HEC2 could be balanced over the two traverse legs using one of the traverse balancing methods given in Chapter 11. From this, the adjusted coordinates of ETLN could be obtained. Since this involves more computation, the simple mean adjustment method in paragraph F-7 is more practical.

#### F-7. Approximate Adjustment of ETLN Using Mean Coordinate Values

The coordinates of station ETLN are then computed by finding the mean of the coordinates as computed forward from each fixed station, using the mean vectors in Table F-3:

$$\begin{aligned}X_{\text{ETLN}} (1) &= X_{\text{ETLE}} + dx_{\text{ETLE-ETLN}} \\ &= 1108314.518 + 98.412 = 1108412.930\end{aligned}$$

$$\begin{aligned}X_{\text{ETLN}} (2) &= X_{\text{HEC2}} + dx_{\text{HEC2-ETLN}} \\ &= 1108302.838 + 110.089 = 1108412.927\end{aligned}$$

(Note the sign of the vector HEC2-ETLN is reversed from that observed -- ETLN-HEC2)

Given the small X-coordinate difference (3 mm), a simple mean adjustment is justified, as opposed to a more rigorous and time-consuming least squares adjustment.

$$\begin{aligned}\text{Mean } X_{\text{ETLN}} &= [ X_{\text{ETLE}} (1) + X_{\text{ETLE}} (2) ] / 2 \\ &= [ 1108412.930 + 1108412.927 ] / 2 \\ &= 1108412.928\end{aligned}$$

The averaged Y and Z coordinates of ETLN are also computed in a manner similar to that for the X:

$$\text{Mean } Y_{\text{ETLN}} = -4856497.985$$

$$\text{Mean } Z_{\text{ETLN}} = 3969892.994$$

#### F-8. Least Squares Adjustment Using FILLNET

To compare the results of this approximate mean adjustment with a least squares solution, all baseline observations from Sessions A and B were input into FILLNET.

Each GPS baseline was given equal relative weighting, as shown. The output from the FILLNET adjustment is shown in Figure F-4 and includes annotations denoting significant statistics resulting from the adjustment. The resultant standard error of unit weight and normalized residuals are significantly below the nominal value of "1.0" indicating that the initial (i.e., a priori) baseline relative weighting ( $\pm 5\text{H}/10\text{V mm} + 2 \text{ ppm}$ ) was somewhat high. None of the normalized residuals exceeded three times the standard error of unit weight ( $\pm 1.95$ ); thus, no observations would be rejected.

The relative line accuracy estimates all exceed 1:10,000; thus the constrained survey meets intended accuracy criteria. Since the FILLNET relative precision estimates are given at the 1-sigma level, they must be divided by 2 to relate to FGCC standards at the 2-sigma (95 percent confidence) level. Thus, the smallest ratio from ETLE to ETLN (1:24,313) is evaluated as 1:12,156 in order to assess compliance with FGCC Third-Order (I) criteria.

The resultant adjusted position of ETLN (in NAD 83 geographical coordinates) from this FILLNET run was:

$$\text{Latitude: } 38^\circ 44' 26.43969''$$

$$\text{Longitude: } 77^\circ 08' 36.34637''$$

These coordinates may then be transformed to X-Y-Z geocentric coordinates using the HP calculator algorithms given in Chapter 11 and then compared with the mean values from the preceding approximate adjustment:

$$\text{L/S } X_{\text{ETLN}} = 1108412.928$$

$$\text{L/S } Y_{\text{ETLN}} = -4856497.986$$

$$\text{L/S } Z_{\text{ETLN}} = 3969893.000$$

PROGRAM FILLNET, Version 3.0.00  
LICENSED TO: ASHTECH INC.

Fillnet Input File jim 38.7 77.1

a = 6378137.000 1/f = 298.2572235 W Longitude positive WEST

PRELIMINARY COORDINATES:

			LAT.		LON.	ELEV.	G.H.	CONSTR.
1	FFF	ETLE	38 44 27.46754	77	8 40.41060	-7.020	0.000	
2		ETLN	38 44 26.61017	77	8 36.40653	8.066	0.000	
3	FFF	HEC2	38 44 36.19465	77	8 39.32344	-5.900	0.000	

GROUP 1, NO. OF VECTORS AND BIAS CONSTRAINTS:

6	0.000	0.001	0.000	0.000	0.000	0.000	0.000	0.000
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VECTORS:

			DX	DY	DZ	LENGTH	ERROR	CODES
ETLE	ETLN		98.418	9.929	-30.837	103.613	5 52.0	102.0 3
ETLN	HEC2		-110.094	159.251	241.448	309.481	5 52.0	102.0 3
ETLE	HEC2		-11.676	169.184	210.602	270.394	5 52.0	102.0 4
ETLE	ETLN		98.405	9.932	-30.834	103.600	5 52.0	102.0 4
ETLE	HEC2		-11.676	169.184	210.602	270.394	5 52.0	102.0 4
ETLN	HEC2		-110.083	159.250	241.444	309.474	5 52.0	102.0 4

SHIFTS:

1	0.000	0.000	0.000
2	-5.258	1.454	-24.857
3	0.000	0.000	0.000

ADJUSTED VECTORS, GROUP 1:

			DX,DY,DZ	V	DN,DE,DU	v	v'
ETLE	ETLN	253 A	98.411	-0.005	-31.750	0.001	0.2
			9.932	0.002	98.145	-0.005	-0.8
			-30.837	-0.001	-9.689	-0.004	-0.3
ETLN	HEC2	253 A	-110.092	0.005	300.849	-0.004	-0.6
			159.251	0.001	-71.760	0.005	0.8
			241.436	-0.004	10.627	-0.002	-0.2
ETLE	HEC2	253 B	-11.680	-0.000	269.099	0.002	0.3
			169.183	-0.001	26.385	-0.001	-0.1
			210.600	0.003	0.938	0.003	0.2
ETLE	ETLN	253 B	98.411	0.008	-31.750	-0.005	-0.7
			9.932	-0.001	98.145	0.007	1.0
			-30.837	-0.004	-9.689	-0.001	-0.1
ETLE	HEC2	253 B	-11.680	-0.000	269.099	0.002	0.3
			169.183	-0.001	26.385	-0.001	-0.1
			210.600	0.003	0.938	0.003	0.2
ETLN	HEC2	253 B	-110.092	-0.006	300.849	0.001	0.2
			159.251	0.002	-71.760	-0.006	-0.8
			241.436	-0.000	10.627	-0.003	-0.2

S.E. OF UNIT WEIGHT = 0.593

Figure F-4. FILLNET least squares adjustment of ETLN (Continued)

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OBS. EQUATIONS	19
UNKNOWN	7
DEGREES OF FREEDOM	12
ITERATIONS	0

HOR. SYSTEM	0.000	3.133	-2.570	-15.571
STD. ERRORS	0.001	3.618	1.807	8.762
XYZ SYSTEM	2.606	2.655	-1.607	

		LAT.		LON.		ELEV.	STD. ERRORS (m)				
1	ETLE	38	44	27.46754	77	8	40.41060	-7.020	0.000	0.000	0.000
2	ETLN	38	44	26.43966	77	8	36.34630	-16.791	0.003	0.003	0.005
3	HEC2	38	44	36.19465	77	8	39.32344	-5.900	0.000	0.000	0.000

		D. LAT.	D. LON.	VERT.
ETLE	ETLN	0.003	0.003	0.005
ETLN	HEC2	0.003	0.003	0.005
ETLE	HEC2	0.000	0.000	0.000
ETLE	ETLN	0.003	0.003	0.005
ETLE	HEC2	0.000	0.000	0.000
ETLN	HEC2	0.003	0.003	0.005

```
*****
****
****          ESTIMATES OF PRECISION          ****
****
****      Based on the VECTOR ACCURACIES produced by      ****
****                      FILLNET                      ****
****
****      This is a reasonable estimate of the accuracies ****
****      of the vectors in the network at 1 SIGMA.      ****
****
*****
```

VECTOR		LENGTH	PPM(h)	RATIO(h)		PPM(v)	RATIO(v)	
ETLE	ETLN	103.607	41.1	1:	24313	48.3	1:	20721
ETLN	HEC2	309.471	13.7	1:	72900	16.2	1:	61894
ETLE	HEC2	270.391	0.0	1:	0	0.0	1:	0
ETLE	ETLN	103.607	41.1	1:	24313	48.3	1:	20721
ETLE	HEC2	270.391	0.0	1:	0	0.0	1:	0
ETLN	HEC2	309.471	13.7	1:	72900	16.2	1:	61894

**Figure F-4. (Concluded)**

Position differences (least squares - mean adjustments):

$$dX = 0.000 \quad dY = 0.001 \quad dZ = 0.006$$

Based on these results, the difference in results between a least squares and simple mean adjustment, for this case, is not significant.

If this were a survey obtained under contract, then a free adjustment would have been used to measure contract performance, not a constrained adjustment. The previous loop/line checks would have adequately served as a free adjustment in checking internal adequacy. Failure of a

constrained survey adjustment to meet minimum relative accuracy standards (presuming the free adjustment did) indicates a problem with the existing network, or connections thereto.

The free adjustment of the same scheme shown in Figure F-5 illustrates the overall improvement in relative accuracy estimates over the constrained adjustments. Although the GPS vector standard errors were decreased from those used in the constrained adjustment, this will have no effect on the relative distance accuracy ratios in a free adjustment. As with the constrained adjustment, the precision ratios must be divided by 2.

# EM 1110-1-1003

1 Aug 96

PROGRAM FILLNET, Version 3.0.00  
LICENSED TO: ASHTECH INC.

Fillnet Input File jim 38.7 77.1

a = 6378137.000 1/f = 298.2572235 W Longitude positive WEST

## PRELIMINARY COORDINATES:

			LAT.		LON.		ELEV.	G.H.	CONSTR.
1	FFF	ETLE	38 44 27.46754	77	8 40.41060		-7.020	0.000	
2		ETLN	38 44 26.61017	77	8 36.40653		8.066	0.000	
3		HEC2	38 44 36.19465	77	8 39.32344		-5.900	0.000	

## GROUP 1, NO. OF VECTORS AND BIAS CONSTRAINTS:

6	0.000	0.001	0.000	0.001	0.000	0.001	0.000	0.001
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## VECTORS:

			DX	DY	DZ	LENGTH	ERROR	CODES	
ETLE	ETLN		98.418	9.929	-30.837	103.613	5 52.0	102.0	3
ETLN	HEC2		-110.094	159.251	241.448	309.481	5 52.0	102.0	3
ETLE	HEC2		-11.676	169.184	210.602	270.394	5 52.0	102.0	4
ETLE	ETLN		98.405	9.932	-30.834	103.600	5 52.0	102.0	4
ETLE	HEC2		-11.676	169.184	210.602	270.394	5 52.0	102.0	4
ETLN	HEC2		-110.083	159.250	241.444	309.474	5 52.0	102.0	4

## SHIFTS:

1	0.000	0.000	0.000
2	-5.259	1.456	-24.857
3	0.004	0.004	0.004

## ADJUSTED VECTORS, GROUP 1:

			DX,DY,DZ	V	DN,DE,DU	v	v'
ETLE	ETLN	253 A	98.413	-0.005	-31.751	0.001	0.2
			9.931	0.002	98.146	-0.005	-0.8
			-30.838	-0.001	-9.690	-0.004	-0.3
ETLN	HEC2	253 A	-110.089	0.005	300.855	-0.004	-0.6
			159.252	0.001	-71.758	0.005	0.8
			241.444	-0.004	10.632	-0.002	-0.2
ETLE	HEC2	253 B	-11.676	-0.000	269.103	0.002	0.3
			169.183	-0.001	26.388	-0.001	-0.1
			210.605	0.003	0.942	0.003	0.2
ETLE	ETLN	253 B	98.413	0.008	-31.751	-0.005	-0.7
			9.931	-0.001	98.146	0.007	1.0
			-30.838	-0.004	-9.690	-0.001	-0.1
ETLE	HEC2	253 B	-11.676	-0.000	269.103	0.002	0.3
			169.183	-0.001	26.388	-0.001	-0.1
			210.605	0.003	0.942	0.003	0.2
ETLN	HEC2	253 B	-110.089	-0.006	300.855	0.001	0.2
			159.252	0.002	-71.758	-0.006	-0.8
			241.444	-0.000	10.632	-0.003	-0.2

S.E. OF UNIT WEIGHT = 0.593

Figure F-5. Free adjustment of network (Continued)

```

NUMBER OF -
OBS. EQUATIONS      22
UNKNOWNNS           10
DEGREES OF FREEDOM  12
ITERATIONS           0

GROUP 1 ROT. ANGLES (sec.) AND SCALE DIFF. (ppm):

HOR. SYSTEM          0.000  0.000  0.000  0.000
STD. ERRORS          0.001  0.001  0.001  0.001
XYZ SYSTEM           0.000  0.000  0.000

```

ADJUSTED POSITIONS:

		LAT.		LON.		ELEV.		STD. ERRORS (m)
1	ETLE	38 44 27.46754	77	8 40.41060		-7.020	0.000 0.000 0.000	
2	ETLN	38 44 26.43961	77	8 36.34626		-16.791	0.002 0.002 0.005	
3	HEC2	38 44 36.19477	77	8 39.32328		-5.896	0.002 0.002 0.005	

ACCURACIES (m):

		D. LAT.	D. LON.	VERT.
ETLE	ETLN	0.002	0.002	0.005
ETLN	HEC2	0.002	0.002	0.005
ETLE	HEC2	0.002	0.002	0.005
ETLE	ETLN	0.002	0.002	0.005
ETLE	HEC2	0.002	0.002	0.005
ETLN	HEC2	0.002	0.002	0.005

```

*****
*****
*****          ESTIMATES OF PRECISION          *****
*****
*****   Based on the VECTOR ACCURACIES produced by   *****
*****                   FILLNET                   *****
*****
*****   This is a reasonable estimate of the accuracies *****
*****   of the vectors in the network at 1 SIGMA.   *****
*****
*****

```

VECTOR	LENGTH	PPM(h)	RATIO(h)	PPM(v)	RATIO(v)
ETLE ETLN	103.608	27.4	1: 36470	48.3	1: 20722
ETLN HEC2	309.477	9.1	1: 109352	16.2	1: 61895
ETLE HEC2	270.395	10.5	1: 95599	18.5	1: 54079
ETLE ETLN	103.608	27.4	1: 36470	48.3	1: 20722
ETLE HEC2	270.395	10.5	1: 95599	18.5	1: 54079
ETLN HEC2	309.477	9.1	1: 109352	16.2	1: 61895

Figure F-5. (Concluded)